DE PERTURBATIONIBUS

QUAS

SATURNUS PER INTEGRAM REVOLUTIONEM IN PALLADEM EXERCET
COMMENTATIO ASTRONOMICA, CUI PALLADIS ORBITAE DETERMINATIO
NOVA SUPERSTRUITUR.

DISSERTATIO

QUAM

AMPLISSIMI PHILOSOPHORUM ORDINIS AUCTORITATE

PRO SUMMIS

IN PHILOSOPHIA HONORIBUS RITE ADIPISCENDIS

DIE XV MENSIS DECEMBRIS MDCCCLX

H. L. Q. C.

PUBLICE DEFENDET

AUCTOR

FRIDERICUS GUILELMUS GÜNTHER

SILESIUS.

OPPONENTIBUS:

RICH. SADEBECK, STUD. PHIL.
AUG. KLINGER, CAND. PHIL.
JOS. DOBROSCHKE, CAND. PHIL.

Breslau

WRATISLAVIAE

TYPIS GRASSII, BARTHII ET SOCII (W. FRIEDRICH).

VIRO

AMPLISSIMO CELEBERRIMO

J. G. GALLE

PHILOSOPHIAE DOCTORI, ASTRONOMIAE IN UNIVERSITATE LITTERARUM VRATISLAVIENSI PROFESSORI PUBLICO ORDINARIO,ORDINIS AQUILAE RUBRAE TERTIAE CLASSIS CUM LEMNISCO ET ORDINIS FRANCOGALLICI DE LEGIONE HONORIS EQUITI ETC.

HASCE PAGELLAS

GRATI ANIMI SIGNUM ESSE VOLUIT

AUCTOR.

In ephemeridibus, Berolini a Celeberrimo Encke editis, pluribus ex annis planetarum Massaliae, Egeriae, Phocaeae, Euterpes, Amphitrites et Uraniae orbitas, eorumque cursum annuum composui. Initio horum calculorum, quum orbitae istorum corporum coelestium nonnisi approximatae essent, solius Jovis perturbationibus rationem tuli, postea vero, orbitis jam in arctiores fines inclusis, etiam Saturni perturbationes addidi, eoque contigit mihi, ut planetae facillime telescopiis inveniri possent, erroresque inter calculum et observationem perparvi fierent. Et re vera contendi potest, et ex hac disquisitione patebit, calculis perturbationum, a Jove et Saturno effectarum orbitae planetae eruendae in longissimum tempus satisfieri, ita ut errores satis parvi tantum relinquantur.

Nihilominus in calculis ephemeridum quatuor planetarum, initio hujus saeculi detectorum, scilicet Junonis, Vestae, Palladis et Cereris per longum tempus solius Jovis perturbationum ratio habita est, quoniam isto tempore observationes corporum coelestium nondum ea, qua nunc, subtilitate instituebantur, et calculi hujusmodi nonnisi a perpaucis astronomis absolvebantur. Haud raro igitur in his planetis propter neglectas Saturni perturbationes majora inventa sunt discrimina inter calculos atque observationes.

Hac re commotus astronomus Celeberrimus Galle, qui huic speculae Vratislaviensi nunc praeest, postquam per quinque jam lustra Palladis planetae orbitam definivit et cursum annuum eruit (conf. ephemerides Berolinenses inde ab anno 1839), aliis muneris negotiis occupatus ideoque impeditus, quominus hoc opus, scilicet definitionem novae Palladis orbitae, respectu habito perturbationum etiam Saturni perficeret, mihi, ut talequid susciperem, initio hujus anni proposuit.

Quonam modo hunc longinquum laborem exantlaverim, in sequentibus breviter exponere liceat mihi. Celeberrimus Galle ante hos duos annos in libello suo: „Ueber die Verbesserung der Planeten-Elemente aus beobachteten Oppositionen, angewandt auf eine neue Bestimmung der Pallas-Bahn“ novam planetae Palladis orbitam, duodecim adhibitis oppo-

sitionibus inde ab anno 1816 usque ad annum 1855 eruit. Haec orbita, cujus elementa erant

i	34°	37′	46″,22
☊	172	33	19,79
φ	14	12	46,45
π	121	14	18,22
μ			770″,74028
L_0	49°	5′	23″,26

relata ad epocham atque aequinoctium medium anni 1810,0 temporis medii Berolinensis, in aequationibus conditionalibus viginti quatuor errores reliquit sequentes:

	1816 *)	1821	1827	1830	1834
dl	— 42″,80	+ 57″,77	+ 19″,06	— 30″,42	+ 12″,84
db	+ 11,34	+ 16,16	— 12,58	— 4,80	+ 16,74
	1836	**1839**	**1841**	**1844**	**1850**
dl	— 5″,95	— 30″,44	+ 1″,74	— 15″,23	— 9″,12
db	— 10,70	— 5,12	+ 0,55	— 12,61	+ 17,37
	1853	**1855**			
dl	+ 46″,46	— 4″,58			
db	— 4,49	+ 40,38			

Hi quidem errores sine dubio praecipue ea de causa satis magni remanserunt, quod in computanda orbita nonnisi perturbationum unius planetae majoris, scilicet Jovis, ratio est habita, actiones contra aliorum, imprimis Saturni neglectae sunt. Magnitudo vero harum Saturni perturbationum, quanta possit evadere, infra exemplo elucebit, ut vix amplius in construendis planetarum ephemeridibus omitti posse videatur.

Considerans, a quo temporis momento calculum perturbationum Saturni inciperem, primum difficile videbatur, usque ad annum 1810 recurrere, quo Jovis perturbationes initium ceperunt. Hae enim, inde ab anno 1815 usque ad 1838 a Cel. Encke, ab 1839 a Cl. Galle erutae, mihi quidem praesto erant (communicante Cl. Galle); deerat autem calculus perturbationum ab anno 1810 usque ad annum 1815 (ut videtur ab illustrissimo Gauss institutus); porro loca Saturni illis temporibus ante annum 1830 multo opere e tabulis elicienda erant, ut igitur tempore, hunc in finem mihi dato, vix ad scopum propositum venire viderer. Quum vero operae pretium sine dubio esset, periculum facere, quomodo calculus perturbationum Saturni longioris temporis intervalli, conjunctus cum Jovis perturbationibus in novam Palladis orbitam inveniendam influeret, et quomodo calculus cum observationibus consentiret, elegi annum 1830,0 tanquam initium perturbationum, quod inde

*) conf. disquisitio Cel. Galle pag. 13 sub finem.

ab hoc anno ephemerides Berolinenses multo accuratiora loca planetarum praebent, ut igitur facillime calculo interpolationis et reductione in orbitam Saturni loca in meum usum convertere possem.

Si l' et b' longitudines latitudinesque, relatae ad aequinoctium medium fixum significant, L' Ω' i' longitudinem in orbita, nodum ascendentem et inclinationem denotant, est secundum Cel. Encke (Ephem. Berol. 1838)

$$\sin\left[\tfrac{1}{2}(l_{,}' + l_0') - \Omega'\right] tg i' = \frac{\sin(b_{,}' + b_0')}{2\cos b_{,}' \cos b_0' \cos\tfrac{1}{2}(l_{,}' - l_0')}$$

$$\cos\left[\tfrac{1}{2}(l_{,}' + l_0') - \Omega'\right] tg i' = \frac{\sin(b_{,}' - b_0')}{2\cos b'_{,} \cos b_0' \sin\tfrac{1}{2}(l_{,}' - l_0')}$$

$$L' = l' + tg\tfrac{1}{2}i'^2 \sin 2(l' - \Omega') + \tfrac{1}{2} tg\tfrac{1}{2} l'^4 \sin 4(l' - \Omega') \ldots\ldots$$

Inventi sunt valores ipsorum i' et Ω'

	i'	Ω'
1830—1835 :	2° 29′ 10″,9	112° 7′ 7″,8
1835—1840 :	2 29 19,6	111 57 5,0
1840—1845 :	2 29 28,7	111 47 3,5
1845—1850 :	2 29 29,3	111 54 2,8
1850—1856 :	2 29 30,4	112 1 40,0
1856—1863 :	2 29 31,0	112 9 36,2

Ope harum quantitatum prodeunt longitudines in orbita Saturni L'; radii vectores elliptici interpolatione simplici eruti sunt. Haec data forsan uni alterive astronomo, calculum similem suscepturo, quod attinet ad planetas Vestam, Junonem et Cererem utilitati alicui erunt, quam ob causam haec loca Saturni omnia hic exhibere non superfluum videtur.

Tempus med. Berol. (intervalla quinquaginta dierum.)*)		L' aequ. med. 1810,0.	log. r' (radius vector.)	Tempus med. Berol.		L' aequ. med. 1810,0.	log. r' (radius vector.)
1829	Oct. 11	130° 15′ 51″,4	0,9612037	1831	Jan. 4	146° 30′ 18″.5	0,9650874
	Nov. 30	132 5 5,1	0,9612061		Febr. 23	148 17 11,8	0,9656922
1830	Jan. 19	133 54 1,9	0,9612241		Apr. 14	150 3 46,8	0,9663084
	Mart. 10	135 42 53,4	0,9617323		Jun. 3	151 50 3,7	0,9669340
	Apr. 29	137 31 29,1	0,9622557		Jul. 23	153 36 1,2	0,9675686
	Jun. 18	139 19 48,8	0,9627944		Spt. 11	155 21 40,3	0,9682144
	Aug. 7	141 7 51,5	0,9633481		Oct. 31	157 7 0,5	0,9688671
	Spt. 26	142 55 37,6	0,9639139		Dec. 20	158 52 1,5	0,9695277
	Nov. 15	144 43 6,5	0,9644930	1832	Febr. 8	160 36 43,2	0,9701948

*) Intervallum idem quinquaginta dierum quod apud Jovem adhibui, quanquam intervalla C dierum jam suffecissent ad satis accuratam integralium definitionem id quod disquisitio posthac instituta docuit.

		L'			log. r'
1832	Mart. 29	162°	21′	5,″5	0,9708687
	Maj. 18	164	5	7,8	0,9715486
	Jul. 7	165	48	50,7	0,9722352
	Aug. 26	167	32	14,6	0,9729254
	Oct. 15	169	15	18,8	0,9736191
	Dec. 4	170	58	2,1	0,9743164
1833	Jan. 23	172	40	26,0	0,9750148
	Mart. 14	174	22	30,7	0,9757148
	Maj. 3	176	4	15,6	0,9764179
	Jun. 22	177	45	40,7	0,9771215
	Aug. 11	179	26	46,3	0,9778256
	Spt. 30	181	7	32,5	0,9785274
	Nov. 19	182	47	58,9	0,9792296
1834	Jan. 8	184	28	7,5	0,9799311
	Febr. 27	186	7	56,6	0,9806291
	Apr. 18	187	47	27,0	0,9813234
	Jun. 7	189	26	38,4	0,9820142
	Jul. 27	191	5	31,0	0,9827013
	Spt. 15	192	44	5,0	0,9833832
	Nov. 4	194	22	21,1	0,9840599
	Dec. 24	196	0	19,1	0,9847303
1835	Febr. 12	197	37	58,4	0,9853960
	Apr. 3	199	15	20,5	0,9860550
	Maj. 23	200	52	26,2	0,9867078
	Jul. 12	202	29	15,2	0,9573515
	Aug. 31	204	5	46,9	0,9879882
	Oct. 20	205	42	1,8	0,9886174
	Dec. 9	207	18	1,4	0,9892359
1836	Jan. 28	208	53	45,6	0,9898475
	Mart. 18	210	29	13,3	0,9904483
	Maj. 7	212	4	25,8	0,9910395
	Jun. 26	213	39	22,7	0,9916210
	Aug. 15	215	14	5,4	0,9921924
	Oct. 4	216	48	34,0	0,9927630
	Nov. 23	218	22	48,0	0,9932997
1837	Jan. 12	219	56	48,3	0,9938356
1837	Mart. 3	221°	30′	35″,8	0,9943608
	Apr. 22	223	4	9,7	0,9948716
	Jun. 11	224	37	31,6	0,9953715
	Jul. 31	226	10	41,0	0,9958597
	Spt. 19	227	43	38,2	0,9963316
	Nov. 8	229	16	24,3	0,9967908
	Dec. 28	230	48	59,1	0,9972370
1838	Febr. 16	232	21	22,5	0,9976706
	Apr. 7	233	53	36,2	0,9980876
	Maj. 27	235	25	40,0	0,9984906
	Jul. 16	236	57	33,5	0,9988798
	Spt. 4	238	29	18,2	0,9992533
	Oct. 24	240	0	54,8	0,9996129
	Dec. 13	241	32	20,7	0,9999566
1839	Febr. 1	243	3	40,2	1,0002851
	Mart. 23	244	34	51,2	1,0005974
	Maj. 12	246	5	55,1	1,0008939
	Jul. 1	247	36	52,7	1,0011737
	Aug. 20	249	7	43,0	1,0014379
	Oct. 9	250	38	27,6	1,0016855
	Nov. 28	252	9	6,5	1,0019171
1840	Jan. 17	253	39	39,5	1,0021310
	Mart. 7	255	10	8,1	1,0023285
	Apr. 26	256	40	32,8	1,0025079
	Jun. 15	258	10	52,2	1,0026706
	Aug. 4	259	41	8,1	1,0028149
	Spt. 23	261	11	21,5	1,0029424
	Nov. 12	262	41	31,3	1,0030513
1841	Jan. 1	264	11	39,1	1,0031424
	Febr. 20	265	41	44,9	1,0032152
	Apr. 11	267	11	49,2	1,0032690
	Maj. 31	268	41	52,3	1,0033052
	Jul. 20	270	11	54,5	1,0033228
	Spt. 8	271	41	56,7	1,0033210
	Oct. 28	273	11	59,4	1,0033014
	Dec. 17	274	42	2,7	1,0032632

		L'			log. r'
1842	Febr. 5	276°	12′	7″,3	1,0032051
	Mart. 27	277	42	13,7	1,0031288
	Maj. 16	279	12	22,1	1,0030339
	Jul. 5	280	42	32,5	1,0029206
	Aug. 24	282	12	47,1	1,0027886
	Oct. 13	283	43	5,1	1,0026390
	Dec. 2	285	13	26,7	1,0024695
1843	Jan. 21	286	43	52,6	1,0022825
	Mart. 12	288	14	23,7	1,0020778
	Maj. 1	289	45	0,2	1,0018557
	Jun. 20	291	15	42,0	1,0016162
	Aug. 9	292	46	30,1	1,0013588
	Spt. 28	294	17	24,8	1,0010845
	Nov. 17	295	48	26,4	1,0007922
1844	Jan. 6	297	19	35,0	1,0004839
	Febr. 25	298	50	51,7	1,0001599
	Apr. 15	300	22	16,6	0,9998186
	Jun. 4	301	53	49,8	0,9994621
	Jul. 24	303	25	31,5	0,9990902
	Spt. 12	304	57	23,4	0,9987012
	Nov. 1	306	29	24,5	0,9982980
	Dec. 21	308	1	35,5	0,9978799
1845	Febr. 9	309	33	56,7	0,9974626
	Mart. 31	311	6	29,2	0,9970157
	Maj. 20	312	39	12,9	0,9965542
	Jul. 9	314	12	7,8	0,9960791
	Aug. 28	315	45	14,7	0,9955901
	Oct. 17	317	18	34,1	0,9950877
	Dec. 6	318	52	6,2	0,9945712
1846	Jan. 25	320	25	51,0	0,9940421
	Mart. 16	321	59	49,5	0,9935011
	Maj. 5	323	34	2,0	0,9929482
	Jun. 24	325	8	27,8	0,9923823
	Aug. 13	326	43	8,5	0,9918052
	Oct. 2	328	18	3,8	0,9912172
	Nov. 21	329	53	14,4	0,9906189

		L'			log. r'
1847	Jan. 10	331°	28′	40″,1	0,9900104
	Mart. 1	333	4	21,4	0,9893919
	Apr. 20	334	40	19,0	0,9887632
	Jun. 9	336	16	32,7	0,9881264
	Jul. 29	337	53	2,8	0,9874813
	Spt. 17	339	29	49,9	0,9868267
	Nov. 6	341	6	54.3	0,9861643
	Dec. 26	342	44	15,9	0,9854955
1848	Febr. 14	344	21	55,2	0,9848204
	Apr. 4	345	59	51,6	0,9841386
	Maj. 24	347	38	6,7	0,9834519
	Jul. 13	349	16	40,3	0,9827592
	Spt. 1	350	55	32,2	0,9820627
	Oct. 21	352	34	43,2	0,9813611
	Dec. 10	354	14	12,6	0,9806557
1849	Jan. 29	355	54	1,7	0,9799465
	Mart. 30	357	34	9,9	0,9792364
	Maj. 9	359	14	37,2	0,9785250
	Jun. 28	0	55	23,8	0,9778119
	Aug. 17	2	36	30,0	0,9771001
	Oct. 6	4	17	55,9	0,9763863
	Nov. 25	5	59	41,4	0,9756740
1850	Jan. 14	7	41	46,5	0,9749623
	Mart. 5	9	24	11,7	0,9742538
	Apr. 24	11	6	56,5	0,9735469
	Jun. 13	12	50	1,3	0,9728433
	Aug. 2	14	33	25,4	0,9721426
	Spt. 21	16	17	9,6	0,9714474
	Nov. 10	18	1	13,8	0,9707585
	Dec. 30	19	45	37,2	0,9700746
1851	Febr. 18	21	30	20,3	0,9693964
	Apr. 9	23	15	22,2	0,9687269
	Maj. 29	25	0	44,6	0,9680652
	Jul. 18	26	46	25,5	0,9674115
	Spt. 6	28	32	25,1	0,9667672
	Oct. 26	30	18	43,8	0,9661334

		L'	log. r'
1851	Dec. 15	32° 5′ 20″,8	0,9655106
1852	Febr. 3	33 52 15,7	0,9648995
	Mart. 24	35 39 28,9	0,9642998
	Maj. 13	37 27 0,0	0,9637129
	Jul. 2	39 14 48,1	0,9631388
	Aug. 21	41 2 53,2	0,9625792
	Oct. 10	42 51 15,4	0,9620339
	Nov. 29	44 39 53,4	0,9615042
1853	Jan. 18	46 28 47,3	0,9609902
	Mart. 9	48 17 56,6	0,9604937
	Apr. 28	50 7 21,2	0,9600149
	Jun. 17	51 57 0,2	0,9595531
	Aug. 6	53 46 53,0	0,9591098
	Spt. 25	55 36 59,6	0,9586857
	Nov. 14	57 27 18,8	0,9582809
1854	Jan. 3	59 17 50,4	0,9578959
	Febr. 22	61 8 33,9	0,9575317
	Apr. 13	62 59 28,9	0,9571881
	Jun. 2	64 50 34,5	0,9568662
	Jul. 22	66 41 49,8	0,9565657
	Spt. 10	68 33 14,7	0,9562874
	Oct. 30	70 24 48,3	0,9560318
	Dec. 19	72 16 29,6	0,9557995
1855	Febr. 7	74 8 18,3	0,9555908
	Mart. 29	76 0 14,1	0,9554053
	Maj. 18	77 52 15,6	0,9552446
	Jul. 7	79 44 22,1	0,9551080
	Aug. 26	81 36 33,1	0,9549956
	Oct. 15	83 28 47,8	0,9549076
	Dec. 4	85 21 5,4	0,9548444
1856	Jan. 23	87 13 25,2	0,9548052
	Mart. 13	89 5 46,8	0,9547910
	Maj. 2	90 58 8,9	0,9548024
	Jun. 21	92 50 31,0	0,9548385
	Aug. 10	94 42 52,1	0,9548983
	Spt. 29	96 35 12,0	0,9549831
1856	Nov. 18	98° 27′ 29″,0	0,9550928
1857	Jan. 7	100 19 43,5	0,9552266
	Febr. 26	102 11 54,0	0,9553841
	Apr. 17	104 4 0,1	0,9555660
	Jun. 6	105 56 0,4	0,9557725
	Jul. 26	107 47 54,6	0,9560022
	Spt. 14	109 39 42,5	0,9562552
	Nov. 3	111 31 22,7	0,9565307
	Dec. 23	113 22 54,3	0,9568279
1858	Febr. 11	115 14 17,0	0,9571476
	Apr. 2	117 5 30,5	0,9574883
	Maj. 22	118 56 33,3	0,9578502
	Jul. 11	120 47 25,3	0,9582335
	Aug. 30	122 38 5,7	0,9586366
	Oct. 19	124 28 34,0	0,9590595
	Dec. 8	126 18 49,7	0,9595099
1859	Jan. 27	128 8 51,7	0,9599599
	Mart. 18	129 58 40,0	0,9604371
	Maj. 7	131 48 13,7	0,9609316
	Jun. 26	133 37 32,1	0,9614428
	Aug. 15	135 26 35,4	0,9619696
	Oct. 4	137 15 22,7	0,9625111
	Nov. 23	139 3 53,4	0,9630674
1860	Jan. 12	140 52 7,2	0,9636377
	Mart. 2	142 40 4,0	0,9642206
	Apr. 21	144 27 43,4	0,9648146
	Jun. 10	146 15 4,5	0,9654201
	Jul. 30	148 2 7,5	0,9660367
	Spt. 18	149 48 52,0	0,9666632
	Nov. 7	151 35 18,0	0,9672984
	Dec. 27	153 21 24,6	0,9679418
1861	Febr. 15	155 7 11,5	0,9685928
	Apr. 6	156 52 39,8	0,9692495
	Maj. 26	158 37 48,9	0,9699126
	Jul. 15	160 22 38,3	0,9705812
	Spt. 3	162 7 8,2	0,9712532

		L'	$\log. r'$
1861	Oct. 23	163° 51′ 18″,4	0,9719293
	Dec. 12	165 35 8,7	0,9726087
1862	Jan. 31	167 18 39,4	0,9732904
	Mart. 22	169 1 50,8	0,9739742
	Maj. 11	170 44 42,5	0,9746582
	Jun. 30	172 27 14,6	0,9753431
	Aug. 19	174 9 27,4	0,9760288
	Oct. 8	175 51 20,8	0,9767135
1862	Nov. 27	177° 32′ 54″,7	0,9773967
1863	Jan. 16	179 14 9,3	0,9780785
	Mart. 7	180 55 5,3	0,9787595
	Apr. 26	182 35 41,9	0,9794371
	Jun. 15	184 15 59,5	0,9801124
	Aug. 4	185 55 58,8	0,9807834
	Spt. 23	187 35 39,5	0,9814509
	Nov. 12	189 15 1,5	0,9831143

Longitudines in orbita, modo exhibitae, ut supra denotatum est, ad aequinoctium medium anni 1810,0 reductae sunt, ob eam maxime causam, ut elementis Cel. Galle, ad idem temporis momentum relatis, neque minus omnibus calculis, qui ab eodem astronomo clarissimo benevolentissime mecum communicati sunt, uti possem.

In computandis Saturni perturbationibus eam secutus sum methodum, quam Celeberrimus Encke in Ephemeridibus Berolinensibus annorum 1837 et 1838 dilucidavit; calculum igitur sic institui, ut quantitates differentiales elementorum $\frac{dL_0}{dt}$, $\frac{d\mu}{dt}$, $\frac{d\pi}{dt}$, $\frac{d\varphi}{dt}$, $\frac{d\Omega}{dt}$ et $\frac{di}{dt}$ elicerem (ubi L_0 longitudinem mediam aequinoctii initialis, μ motum medium sidereum unius diei, π longitudinem perihelii, φ angulum excentricitatis ellipticae, Ω longitudinem nodi ascendentis, i inclinationem orbitae ad eclipticam, auctore illustr. Gauss in opere: „Theoria motus corporum coelestium“ significant.

Ad integrationem efficiendam quantitates constantes addendae eo modo definiebantur, ut integralia omnia pro initio anni 1830 (temporis medii Berolinensis) evanescerent. Duodecim vero Palladis oppositiones, scilicet annorum 1830, 1834, 1836, 1839, 1841, 1844, 1850, 1853, 1855, 1857, 1858, 1859 in novum calculum introduxi, et massa Saturni, toti calculo tanquam fundamento inserviente, Besseliana, scilicet $\frac{1}{3501,6}$ usus sum, si solis massa aequalis ponitur unitati.

Pro eruendis oppositionibus annorum 1857, 1858 et 1859 sequentes inveni observationes earumque differentias a calculis Celeberrimi Galle in Ephemeridibus Berolinensibus.

1857

C — O (Calculus minus observatio).

	$\triangle\alpha$	$\triangle\delta$	Locus observ.	Fons.
Jan. 15	— $9^s,08$	— 39″,9	Grenovici	Annales Grenov. 1857
„ 23	— 8,67	— 39,0	„	„ „ 1857
„ 27	— 8,74	— 36,1	„	„ „ 1857

C — O

	Δα	Δδ	Locus observ.	Fons.
Febr. 8	— 9s,30	— 53″,6	Kremifani	Ann. astr. Alton. No. 1104
„ 10	— 8,24	— 40,3	Grenovici	Ann. Grenov. anni 1857
„ 12	— 8,06	— 40,1	„	„

1858

	Δα	Δδ	Locus observ.	Fons.	
Maj. 29	— 1s,40	— 22″,4	Regiomonti Pr.	Annal. Alton.	No. 1194
„ 30	— 1,33	— 20,7	„	„	
Jun. 1	— 1,43	— 22,2	„	„	
„ 3	— 1,38	— 21,3	„	„	
„ 4	— 1,58	— 21,8	„	„	
„ 5	— 1,14	— 21,0	„	„	
„ 5	— 1,26	— 24,2	Kremifani	„	No. 1182
„ 6	— 1,24	— 24,8	Regiomonti Pr.	„	„ 1194
„ 7	— 1,21	— 21,0	Kremifani	„	„ 1182
„ 9	— 1,31	— 24,6	„	„	„ 1182

1859

	Δα	Δδ	Locus observ.	Fons.	
Jul. 25	— 2s,46	+ 33″,6	Gottingae	Annal. Alton.	No. 1215
„ 29	— 2,23	+ 32,4	Kremifani	„	„ 1229
„ 31	— 2,01	+ 28,7	„	„	„ 1229
Aug. 3	— 2,06	+ 30,5	„	„	„ 1229
„ 3	— 2,50	+ 30,7	Gottingae	„	„ 1215
„ 3	— 2,18	+ 30,0	„	„	„ 1215
„ 5	— 2,45	+ 28,6	„	„	„ 1215
„ 5	— 2,58	+ 25,4	„	„	„ 1215
„ 6	— 2,27		„	„	„ 1215
„ 7	— 2,22	+ 29,2	Viennae	„	„ 1228
„ 7	— 2,18	+ 30,4	Kremifani	„	„ 1229
„ 8	— 1,99	+ 28,5	„	„	„ 1229
„ 8	— 2,16	+ 31,7	Viennae	„	„ 1228
„ 9	— 2,57	+ 32,2	„	„	„ 1228
„ 24	— 2,12	+ 34,1	Kremifani	„	„ 1229
„ 25	— 1,94	+ 36,2	„	„	„ 1229

Ad eruendas oppositiones annorum 1857, 1858, 1859 praecepta disquisitionis Cel. Galle secutus, momenta calculo posteriori superstruenda haec accepi:

Annus.	☿ ☍ ☉ Temp.Med. Berol.	☿ Longitudo geocentrica. Aequin. med.	☿ Latitudo geocentrica.	☉ Log. radii vectoris.
1830	Apr. 26, 77184	216° 26′ 34″,34	+ 38° 53′ 1″,30	0,0031414
1834	Febr. 8, 03579	139 19 17,06	— 37 2 25,55	9,9942973
1836	Aug. 13, 97002	321 35 40,60	+ 27 29 13,00	0,0053827
1839	Mart. 31, 77409	190 49 29,61	+ 20 21 51,03	9,9999504
1841	Sept. 4, 24717	341 56 7,33	+ 10 37 7,54	0,0033306
1844	Maj. 10, 77336	230 35 25,25	+ 44 30 2,66	0,0046183
1850	Aug. 22, 83606	329 45 33,94	+ 21 24 35,01	0,0046148
1853	Apr. 17, 11294	207 27 31,34	+ 33 42 17,23	0,0020285
1855	Sept. 13, 46789	350 32 1,38	+ 2 7 24,03	0,0023860
1857	Jan. 22, 54153	123 0 32,80	— 48 11 33,00	9,9932113
1858	Maj. 26, 79395	245 37 13,78	+ 47 47 48,90	0,0058720
1859	Aug. 10, 51758	317 44 31,08	+ 30 11 48,25	0,0057069

Hisce temporibus respondentes valores perturbationum inde ab anno 1830,0 et Jovis et Saturni enumerare haud abs re videtur, ut pelluceat, quanta sit discrepantia inter actiones amborum planetarum (massa Jovis, qua Cel. Galle quoque in calculis usus est, secundum Clar. Bessel $\frac{1}{1047,879}$ posita).

Jupiter.

Annus.	$\int \frac{di}{dt}$	$\int \frac{d\Omega}{dt}$	$\int \frac{d\varphi}{dt}$	$\int \frac{d\pi}{dt}$	$\int \frac{d\mu}{dt}$	$\int \frac{dL_0}{dt} + \iint \frac{d^2\mu}{dt^2}$
1830	— 0′ 4″,03	— 0′ 14″,96	— 0′ 22″,54	— 0′ 7″,55	—0″,02709	— 0° 7′ 9″,83
1834	+ 2 36,05	— 6 40,85	— 14 43,28	+26 24,54	—0,64413	— 1 18 21,15
1836	+ 2 31,94	— 8 24,85	— 13 44,19	+32 29,15	—1,07372	— 1 58 13,33
1839	+ 2 44,21	— 8 30,42	— 12 20,08	+39 35,43	—1,15474	— 2 36 41,55
1841	+ 2 18,20	— 10 1,93	— 10 53,00	+24 29,61	—0,36975	— 3 5 0,18
1844	+ 2 16,62	— 10 15,87	— 10 9,86	+14 51,10	—0,56852	— 3 39 50,85
1850	+ 2 16,22	— 12 32,34	— 10 50,17	+ 4 7,61	—0,84662	— 4 46 30,18
1853	+ 1 56,93	— 13 0,29	— 12 26,63	+ 7 4,86	—0,71070	— 5 23 29,54
1855	+ 6 41,30	— 23 17,49	— 13 48,60	+ 25 34,06	—0,72456	— 6 8 11,91

Jupiter.

Annus.	$\int \frac{di}{dt}$	$\int \frac{d\Omega}{dt}$	$\int \frac{d\varphi}{dt}$	$\int \frac{d\pi}{dt}$	$\int \frac{d\mu}{dt}$	$\int \frac{dL_0}{dt} + \iint \frac{d^2\mu}{dt^2}$
1857	+ 7′ 22″,97	— 23′ 3″,09	— 13′ 18″,59	+42′55″,93	+0″,53531	— 6° 17′ 49″,85
1858	+ 7 5,03	— 23 27,09	— 12 10,58	+43 54,99	+0,18186	— 6 28 29,63
1859	+ 7 20,43	— 24 42,33	— 11 15,68	+44 53,06	+0,35697	— 6 38 56,07

Saturnus.

Annus.	$\int \frac{di}{dt}$	$\int \frac{d\Omega}{dt}$	$\int \frac{d\varphi}{dt}$	$\int \frac{d\pi}{dt}$	$\int \frac{d\mu}{dt}$	$\int \frac{dL_0}{dt} + \iint \frac{d^2\mu}{dt^2}$
1830	+ 0″,15	+ 0″,17	— 0″,78	— 3″,30	+0″,00678	+ 1″,52
1834	+ 0,96	— 2,09	— 10,24	+ 29,72	—0,00694	— 20,25
1836	— 0,24	— 3,58	— 6,09	— 2,30	+0,00113	— 26,56
1839	— 1,96	— 4,93	— 11,28	— 9,62	—0,00837	— 37,97
1841	— 0,87	— 17,52	— 3,68	— 2,75	+0,00257	— 53,52
1844	— 0,48	— 19,04	— 3,63	— 19,53	+0,00175	— 54,70
1850	+ 5,24	— 27,17	— 11,26	+ 0,12	—0,00332	— 1′ 20,11
1853	+ 1,33	— 29,71	— 19,58	+ 1,07	—0,02172	— 1 46,30
1855	+ 2,83	— 36,55	— 10,59	+ 2,63	+0,00696	— 2 1,70
1857	+ 2,65	— 39,58	— 8,03	— 10,77	—0,01173	— 2 1,06
1858	+ 3,10	— 41,19	— 11,45	— 6,77	—0,00482	— 1 59,54
1859	+ 3,88	— 45,00	— 14,62	+ 9,11	—0,02601	— 2 9,74

Hisce calculis absolutis, investigandum erat, quemadmodum perturbationes Jovis et Saturni conjunctae in loca geocentrica temporibus oppositionum, antea denotatis, influerent. Id quod duplici modo fieri poterat. Primum enim effectus Saturni solius per valores integrales, modo exhibitos, conjunctim cum quotientibus differentialibus elementorum secundum formulas has eruitur:

$$dl = \frac{dl}{dL_0}\left(\int \frac{dL_0}{dt} + \iint \frac{d^2\mu}{dt^2}\right) + \frac{dl}{d\pi}\int \frac{d\pi}{dt} + \frac{dl}{d\varphi}\int \frac{d\varphi}{dt} + \frac{dl}{d\,\Omega}\int \frac{d\,\Omega}{dt} + \frac{dl}{di}\int \frac{di}{dt}$$

et

$$db = \frac{db}{dL_0}\left(\int \frac{dL_0}{dt} + \iint \frac{d^2\mu}{dt^2}\right) + \frac{db}{d\mu}\int \frac{d\mu}{dt} + \frac{db}{d\pi}\int \frac{d\pi}{dt} + \frac{db}{d\varphi}\int \frac{d\varphi}{dt} + \frac{db}{d\,\Omega}\int \frac{d\,\Omega}{dt} + \frac{db}{di}\int \frac{di}{dt}$$

His formulis adhibitis meri perturbationum valores Saturni in loca singula geocentrica planetae Palladis inveniuntur, qui cum jam prius constitutis, per Jovem effectis valoribus conjuncti, novas problemati solvendo quantitates suppeditant.

Altera calculandi methodus, quam in hoc opusculo secutus sum, haec est:

Adduntur perturbationes Saturni valoribus pro Jove inventis, et denuo loca geocentrica Palladis temporibus oppositionum respondentia secundum leges a Cel. Galle in disquisitione sua penitus dilucidatas directo calculo derivantur, quo absoluto discrimina observationum calculorumque extemplo prodeunt.

Liceat exemplo, ab oppositione anni 1857 sumto, rem totam paullulum illustrare.

E tabula praecedenti perturbationum sequuntur actiones conjunctae planetarum Jovis et Saturni, sola additione valorum $\int\frac{di}{dt}$, $\int\frac{d\Omega}{dt}$ etc.:

$$\int\frac{di}{dt} = +\ 7'\ 25'',6,\quad \int\frac{d\Omega}{dt} = -23'\ 42'',7,\quad \int\frac{d\pi}{dt} = +\ 42'\ 45'',2$$

$$\int\frac{d\varphi}{dt} = -13'\ 26'',6,\quad \int\frac{d\mu}{dt} = +\ 0'',52358,\quad \int\frac{dL_0}{dt} + \iint\frac{d^2\mu}{dt^2} = -6^0\ 19'\ 50'',9.$$

Praecessio aequinoctiorum, elementis L_0, π et Ω adjicienda, est $+\ 22'\ 39'',4$, et effectus obliquitatis eclipticae variabilis diminuit inclinationem i 23″,0, addit longitudini nodi ascendentis 0″,8, et diminuit longitudinem perihelii et longitudinem mediam 0″,2. Elementa orbitae primaria, quibus totus nititur calculus, haec sunt

i	34 ^{0}	35′	36″,7
Ω	172	39	13,4
φ	14	3	21,5
π	121	0	22,9
μ			769″,29265
L_0	169 ^{0}	5′	36″,5

relata ad aequinoctium medium anni 1830,0, et correctionibus omnibus, modo citatis, iis applicatis, in sequentia transeunt:

i	34 ^{0}	42′	39″,3
Ω	172	38	10,9
φ	13	49	54,9
π	122	5	47,4
μ			769″,81623
L_0	82^0	25′	21″,0

reducta ad aequinoctium medium 1857 Jan. 22.

Ad exemplum paginae 16 disquisitionis Cel. Galle jam saepius commemoratae, eruuntur valores errorum

$$dl = -592'',83$$
$$db = -117'',20$$

quantitates plane consentientes cum illis, quas prima usi methodo, ut examini subjiceremus calculum, invenimus.

Hicce calculus, per omnes XII oppositiones confectus, sequens errorum (nunc Saturni causa multo majorum) suppeditat systema:

	dl	db
1830 :	— 48'',06	— 24'',80
1834 :	— 147,08	— 57,85
1836 :	— 54,82	+ 19,70
1839 :	— 155,14	— 94,43
1841 :	— 74,70	+ 48,96
1844 :	— 151,91	— 43,16
1850 :	— 102,51	+ 63,09
1853 :	— 246,26	— 115,23
1855 :	— 147,50	+ 119,37
1857 :	— 592,83	— 117,20
1858 :	— 233,91	— 28,70
1859 :	— 131,69	+ 61,55

Quotientibus differentialibus penitus ad normam formularum 14 pag. 6 disquisitionis Cel. Galle computatis, non inutile videtur adnotare, quantitatem t pro initio anni 1830,0 aequalem 0 positam esse, quotientes igitur $\frac{dl}{d\mu} = t\,\frac{dl}{dL_0}$, et $\frac{db}{d\mu} = EI + t\,\frac{db}{dL_0}$, tanquam functiones ipsius t in primis novem oppositionibus diversos esse ab istis, qui ab initio anni 1810 exordium capiunt, quotientes contra $\frac{dl}{dL_0}$, $\frac{dl}{d\pi}$, $\frac{dl}{d\varphi}$, $\frac{dl}{d\Omega}$, $\frac{dl}{di}$ et $\frac{db}{dL_0}$, $\frac{db}{d\pi}$, $\frac{db}{d\varphi}$, $\frac{db}{d\Omega}$, $\frac{db}{di}$, tempori non subjectos, sed e solis elementis nascentes, eosdem mansisse. Ceterum totus hic calculus quotientium differentialium examini subjiciebatur usitato. Instituebatur scilicet cum elementis, ad libitum mutatis, novus locorum geocentricorum computus, cujus a pristino discrimina cum quantitatibus per quotientes differentiales productis comparata sunt.

Propius jam ad problematis nostri solutionem accedentes, aequationes conditionales 24 inter quotientes differentiales, valores errorum et correctiones adhuc incognitas elementorum orbitae ipsorum proferemus, in quibus $d\mu' = 10000\ d\mu$ positum est.

$$0 = -\ 48'',06 + 1,70169\,dL_0 + 0,01987\,d\mu' + 0,00608\,d\pi + 3,37639\,d\varphi - 0,01800\,d\Omega - 0,58209\,di$$
$$0 = -147,08 + 3,11107\ \text{-}\ + 0,46729\ \text{-}\ - 1,23724\ \text{-}\ + 0,93348\ \text{-}\ + 0,11743\ \text{-}\ + 0,63099\ \text{-}$$
$$0 = -\ 54,82 + 0,87526\ \text{-}\ + 0,21164\ \text{-}\ + 0,48008\ \text{-}\ - 0,63106\ \text{-}\ + 0,10667\ \text{-}\ + 0,44594\ \text{-}$$
$$0 = -115,14 + 1,85978\ \text{-}\ + 0,62819\ \text{-}\ - 0,37737\ \text{-}\ + 3,01807\ \text{-}\ + 0,23934\ \text{-}\ - 0,35247\ \text{-}$$
$$0 = -\ 74,70 + 0,87916\,dL_0 + 0,37498\,d\mu' + 0,34290\,d\pi - 1,40742\,d\varphi + 0,23893\,d\Omega + 0,18438\,di$$

$0 = -151'',91$	$+1,65200\, dL_0$	$+0,86644\, d\mu'$	$+0,20287\, d\pi$	$+3,32300\, d\varphi$	$-0,17542\, d\Omega$	$-0,52208\, di$
$0 = -102,51$	$+0,85658$ -	$+0,64584$ -	$+0,42802$ -	$-0,96248$ -	$+0,17070$ -	$+0,36113$ -
$0 = -246,26$	$+1,74076$ -	$+1,48103$ -	$-0,12883$ -	$+3,30977$ -	$+0,08476$ -	$-0,54821$ -
$0 = -147,50$	$+0,93944$ -	$+0,88190$ -	$+0,27675$ -	$-1,72104$ -	$+0,26215$ -	$+0,03746$ -
$0 = -592,83$	$+3,73075$ -	$+3,68775$ -	$-1,50365$ -	$-0,40565$ -	$-0,10851$ -	$+0,72408$ -
$0 = -233,91$	$+1,58354$ -	$+1,64274$ -	$+0,38528$ -	$+3,03340$ -	$-0,30385$ -	$-0,32315$ -
$0 = -131,69$	$+0,89112$ -	$+0,96372$ -	$+0,50569$ -	$-0,44728$ -	$+0,07157$ -	$+0,47739$ -
$0 = -\ 24,80$	$+0,49523$ -	$+0,01181$ -	$+0,08597$ -	$+1,08595$ -	$-0,58120$ -	$+0,98112$ -
$0 = -\ 57,85$	$+1,32009$ -	$+0,18976$ -	$-0,54497$ -	$-0,20470$ -	$-0,77512$ -	$-0,89532$ -
$0 = +\ 19,70$	$-0,41917$ -	$-0,09797$ -	$-0,24494$ -	$+0,16437$ -	$+0,66411$ -	$+0,70265$ -
$0 = -\ 94,43$	$+1,04434$ -	$+0,35696$ -	$-0,16172$ -	$+1,87461$ -	$-0,88262$ -	$+0,50983$ -
$0 = +\ 48,96$	$-0,56955$ -	$-0,24143$ -	$-0,23739$ -	$+0,87437$ -	$+0,80694$ -	$+0,26882$ -
$0 = -\ 43,16$	$+0,28840$ -	$+0,15734$ -	$+0,11796$ -	$+0,59991$ -	$-0,40636$ -	$+1,14000$ -
$0 = +\ 63,09$	$-0,47352$ -	$-0,35426$ -	$-0,25600$ -	$+0,43289$ -	$+0,72952$ -	$+0,54449$ -
$0 = -115,23$	$+0,66410$ -	$+0,57076$ -	$+0,02826$ -	$+1,41363$ -	$-0,69236$ -	$+0,84378$ -
$0 = +119,37$	$-0,64639$ -	$-0,60647$ -	$-0,19421$ -	$+1,17922$ -	$+0,84060$ -	$+0,05411$ -
$0 = -117,20$	$+0,91008$ -	$+0,88965$ -	$-0,35833$ -	$-0,80714$ -	$-0,55175$ -	$-1,14092$ -
$0 = -\ 28,70$	$+0,11472$ -	$+0,12492$ -	$+0,10226$ -	$+0,16087$ -	$-0,21698$ -	$+1,24306$ -
$0 = +\ 61,55$	$-0,39490\, dL_0$	$-0,42343\, d\mu'$	$-0,23534\, d\pi$	$+0,04404\, d\varphi$	$+0,63024\, d\Omega$	$+0,77278\, di$

Aequationes praecedentes secundum methodum minimorum quadratorum solutae has praebent sex aequationes finales, si $dL_0 = x$, $d\mu' = y$, $d\pi = z$, $d\varphi = t$, $d\Omega = u$, $di = v$ designamus:

$$x + 0,59619\, y - 0,17033\, z + 0,52689\, t - 0,09812\, u + 0,01662\, v - 106,4634 = 0$$
$$y + 0,04432\, z - 0,67823\, t - 0,01902\, u + 0,06814\, v - 127,2382 = 0$$
$$z + 0,47927\, t - 0,21690\, u - 0,11138\, v + 3,6956 = 0$$
$$t + 0,01136\, u - 0,04636\, v - 0,4061 = 0$$
$$u + 0,16910\, v + 6,3521 = 0$$
$$v - 9,8048 = 0$$

Inverso nunc ordine quantitatum incognitarum, et repetita aequationum solutione has inveni aequationes finales:

$$v + 0,07832\, u - 0,23125\, t - 0,05449\, z + 0,08773\, y + 0,07239\, x - 22,4347 = 0$$
$$u - 0,35601\, t - 0,01804\, z - 0,52355\, y - 0,82712\, x + 98,3200 = 0$$
$$t - 0,03825\, z + 0,13696\, y + 0,34882\, x - 28,4440 = 0$$
$$z - 0,74467\, y - 1,29603\, x + 13,6272 = 0$$
$$y + 0,95328\, x - 154,2655 = 0$$
$$x - 28,3180 = 0$$

Medio arithmetico ambarum solutionum sumto, fiunt igitur denique elementorum orbitae correctiones:

$$di = +\ 9'',80505$$
$$d\Omega = -\ 8,01110$$
$$d\varphi = +\ 0,95185$$
$$d\pi = -\ 4,79695$$
$$d\mu = +\ 0,01273$$
$$L_0 = +\ 28,31785$$

Quae additae elementis initialibus hanc novam Palladis orbitam praebent:

1830 Jan. 0,0 Temp. Med. Berol.

$$i = 34^0\ 35'\ 46'',51$$
$$\Omega = 172\ \ 39\ \ \ 5,44$$
$$\varphi = 14\ \ \ 3\ \ 22,49$$
$$\pi = 121\ \ \ 0\ \ 18,08$$
$$\mu = 770'',75158$$
$$L_0 = 169^0\ \ 6'\ \ \ 4'',83$$

omnes longitudines, nimirum Ω, π et L_0, relatae ad aequinoctium medium epochae superscriptae.

Computus ponderum horum elementorum plane ad leges, quas Cel. Encke praescripsit (Ephem. Berol. 1836, pag. 266, 267) institutus, valores exhibuit hos pro pondere singulorum elementorum:

i	10,64
Ω	5,08
φ	51,09
π	4,10
μ	6,87
L_0	9,67

Ut totus calculus praecedens ultimo subjiciatur examini, et quidem duplici, derivata sunt duodecim loca oppositionum planetae e novis, modo inventis elementis, cui calculo logarithmi sex decimalium inserviebant.

Errores remanentes in his XII locis sequentes calculo absoluto prodibant:

	1830	**1834**	**1836**	**1839**	**1841**	**1844**
dl	— 0'',40	+ 12'',62	— 2'',49	— 22'',90	— 5'',45	+ 4'',11
db	+ 6'',40	+ 3'',05	— 1'',40	— 5'',23	+ 0'',16	— 0'',76

	1850	**1853**	**1855**	**1857**	**1858**	**1859**
dl	+ 2'',71	— 9'',57	— 12'',54	— 2'',33	+ 19'',85	+ 18'',08
db	+ 5'',89	— 7'',93	+ 18'',17	+ 16'',50	+ 3'',70	+ 0'',15

Valores eruti quantitatum x, y, z, t, u, v porro substituti in XXIV aequationes conditionales, antea prolatas, haec vero discrimina inter observationes calculosque reliquerunt:

	1830	1834	1836	1839	1841	1844
dl	+ 0″,27	+ 12″,56	— 2″,47	— 23″,23	— 5″,15	+ 3″,62
db	+ 5″,62	+ 3″,53	— 1″,83	— 5″,50	+ 0″,25	— 0″,53

	1850	1853	1855	1857	1858	1859
dl	+ 3″,15	— 10″,75	— 13″,35	— 3″,04	+ 20″,31	+ 17″,45
db	+ 5″,73	— 8″,75	+ 18″,73	+ 15″,98	+ 4″,04	+ 0″,18

Unde patet, gravius vitium calculo toti non amplius inhaerere posse. Commemoratu dignum videtur, summam quadratorum errorum, quae ante solutionem aequationum conditionalium 663466″ fuerat, solutione ipsa usque ad 2610″ diminutam esse. Substitutionis examine finito 2583″, calculo denique directo 2513″ errorum residuorum summa quadratorum inventa est.

Quod attinet ad magnitudinem horum numerorum, negari omnino non poterit, errores nunc, adhibitis Saturni perturbationibus, multo minores esse factos, atque hac nova Palladis orbita locum planetae in multos annos derivari posse, differentiam minuti primi dimidiati vix excedentem, errores denique adhuc residuos maximam partem Martis perturbationibus esse adscribendos, quibus hic ea de causa supersedere potuimus, quod in postero tempore, si cursus Palladis tabulis superstruetur, perturbationes generales omnium planetarum majorum calculo subjiciendae erunt. Simul patet, ea, quae hic de perturbationibus disseruimus, quas in Palladem Saturnus per integram revolutionem triginta annorum exercet, ad alios quoque planetas minores adhiberi posse, ita ut calculus perturbationum Jovis et Saturni sufficere videatur, quoties de facili planetarum investigatione et observatione tantummodo agitur.